

Asymptotic freedom and IR freezing in QCD: the role of gluon paramagnetism

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Paramagnetism of gluons is shown to play the basic role in establishing main properties of QCD: IR freezing and asymptotic freedom (AF). Starting with Polyakov background field approach the first terms of background perturbation theory are calculated and shown to ensure not only the classical result of AF but also IR freezing. For the latter only the confining property of the background is needed, and the effective mass entering the IR freezing logarithms is calculated in good agreement with phenomenology and lattice data.

1. INTRODUCTION

The notion of asymptotic Freedom (AF) is basic in establishing QCD as a selfconsistent theory [1]. The extrapolation of the QCD coupling constant $\alpha_s(Q)$ to larger distances (smaller momenta Q) leads however to inconsistencies of several kinds in the pure (nonbackground) perturbation theory:

1. The appearance of Landau ghost pole (and other singularities in higher orders) precludes extrapolation to small Q [2].
2. IR renormalons make the whole perturbation series not summable even in the Borel sense [3].
3. The treatment of perturbation series in the Minkowski space-time has difficulties and should be reformulated [4].

At the same time the IR behavior of α_s in experiment [5] and on the lattice [6] does not show irregularities in the Euclidean region, $Q^2 \geq 0$, and is compatible with IR freezing.

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To ensure this nonsingular behavior a special type of theory was suggested [7], eliminating Landau ghost pole from the beginning, which is phenomenologically successful [8].

To understand what dynamical mechanism makes QCD perturbation theory consistent and brings in the IR freezing, as seen on lattice and experiment, the Background Perturbation Theory (BPTh), formulated earlier in [9], was considered, treating background as a strong collective field with the property of confinement[10].

It was shown in [11], that the basic effect of this confining background is to make $\alpha_s(Q^2)$ finite at all $Q^2 \geq 0$ and thus precluding appearance of Landau ghost pole and IR renormalons. Moreover, the AF logarithm approximately keeps its form at large Q^2 , $\ln \frac{Q^2}{\Lambda_{\text{QCD}}^2}$, but at small Q^2 the argument acquires the additional term, which looks like the two-gluon mass M_{2g}^2 yielding $\ln \left| \frac{Q^2 + M_{2g}^2}{\Lambda_{\text{QCD}}^2} \right|$. This type of form was suggested before [12, 13], however in QCD the appearance of gluon mass is forbidden by gauge invariance. As will be seen the term M_{2g} actually has the meaning of the two-gluon mass, where gluons are connected by the adjoint string. In [11] the IR freezing was considered in the framework of the static $Q\bar{Q}$ potential, and the exact general form of IR behavior and exact value of M_{2g} were not actually given. In the present paper we present a more general derivation of the IR freezing based on the Polyakov background approach [14], where the basic one-loop element is the scalar self-energy (gluon loop) operator $\Pi(Q^2)$. As will be shown, in the confining background $\Pi(Q^2)$ acquires the two-gluon mass $M_{2g} \approx 2$ GeV, and ensures both AF at large Q^2 and IR freezing at small Q^2 . The explicit value of this mass is estimated and it is shown, that numerically IR freezing is not universal: the IR freezing behavior and mass (to be called IR mass) depends on the embedding process. For comparison the background perturbation theory for the static $Q\bar{Q}$ system is considered in the one-loop approximation and it is shown, that the corresponding IR mass is much lower, $M_{2g}^{Q\bar{Q}} \approx 1$ GeV. This latter value is in good agreement with phenomenological description of IR freezing [15, 16] as well as with lattice determinations of α_s [17, 18]. The generalization can be considered as well, leading to the inclusion of multigluon states in the asymptotics of a corresponding Green's function, which coupled by confinement. Thus all theory becomes finite and devoid of IR renormalons [11, 19], hence well defined in the Euclidean region.

To go beyond Euclidean region, one needs to define better the singularity structure of the perturbative series. The logarithmic singularities of the free PTh are not physical as well as those in BPTh. To simplify matter one can take the limit $N_c \rightarrow \infty$, where all QCD

amplitudes contain only poles [20].

The corresponding extrapolation was done in [21, 22] where it was shown, that equidistant mass squared spectra of hadrons allow to replace all logs by Euler ψ -functions, and thus obtain for finite $Q^2 < 0$ simple poles in Minkovskian region, while for large $Q^2 > 0$ in Euclidean region one has standard logarithmic terms. The whole scheme works nicely for both $\beta(\alpha_s)$ and α_s and agrees well both with lattice and phenomenology [21, 22].

In all these considerations the nonpositive definiteness of the β -function of $SU(N_c)$ theory is crucial, and the latter is due to gluon paramagnetic terms in Lagrangian and gluon Green's function. The plan of the paper is as follows. Section 2 is devoted to the extrapolation of the Polyakov method to the IR region. Section 3 contains similar treatment for the static potential $Q\bar{Q}$ system. In section 4 summary and discussion of results is given.

2. ONE-LOOP EVOLUTION OF α_s BY THE POLYAKOV METHOD

As in [14], one starts with the gluonic action $S = \frac{1}{4g^2} \int F_{\mu\nu}^a F_{\mu\nu}^a d^4x$, defined at the scale R_1 (momentum scale $\lambda_1 = 1/R_1$) and consider Wilson transformation to the scale $R_2(\lambda_2)$, which can be considered as the change of the effective integral volume in S from R_1^4 to R_2^4 . Separating gluon field into valence gluons a_μ and background B_μ .

$$A_\mu = a_\mu + B_\mu, \quad (1)$$

one can expand in a_μ , keeping quadratic in a_μ terms

$$F_{\mu\nu}^a F_{\mu\nu}^a = a_\mu^a \left((D_\lambda^2)_{ab} a_\nu^b - 2g F_{\mu\nu}^a(B) a_\mu^b a_\nu^c f^{abc} \right). \quad (2)$$

As Polyakov mentions, the first term, proportional to D_λ^2 , gives rise to diamagnetic interaction of valence gluon with background, $ga_\mu^a B_\lambda^b \partial_\lambda a_\nu^c f^{abc}$, while the second term is paramagnetic interaction of gluon spins with background. Both can be expressed in second order through the scalar gluon self-energy $\Pi(x-y)$, which corresponds to the loop diagram of two massless scalars, and in case of no background is

$$\Pi_0(x) = G_0^2(x) = \frac{1}{(2\pi)^4 x^4}. \quad (3)$$

The resulting expression for the change of one-loop correction from the scale R_1 to the scale R_2 is

$$\frac{1}{g^2(R_2)} = \frac{1}{g^2(R_1)} + \frac{\bar{b}_0}{4\pi} f(R_1, R_2), \quad \bar{b}_0 = \frac{11}{3} N_c \quad (4)$$

where we have defined

$$f(R_1, R_2) = \int_{|x-y|=R_1}^{|x-y|=R_2} d^4(x-y) \Pi(x-y) \quad (5)$$

which yields the standard expression in the free case (no background),

$$f_0(R_1, R_2) = -\frac{1}{4\pi} \ln \frac{R_2}{R_1}. \quad (6)$$

It is this behavior, which produces AF at small R_i and Landau ghost pole [2] appears when the r.h.s. of (4) vanishes. In case of nonzero background it is necessary to take into account, that “scalar gluon” propagator in background, $G(x)$ is no more free and massless. Moreover, if one takes into account confinement, then the product $G^2(x)$ should be replaced by the two-gluon white Green’s function, i.e. the two-gluon glueball Green’s function. $G_{2g}(x)$, and the resulting evolution function

$$f(R_1, R_2) \rightarrow f_{2g}(R_1, R_2),$$

$$f_{2g}(R_1, R_2) \sim \int_{|x|=R_1}^{|x|=R_2} d^4x \Pi_{2g}(x). \quad (7)$$

It is our purpose below in this section to calculate f_{2g} both in coordinate and in the momentum space, proving the IR freezing in an explicit way.

To proceed we shall use the exact Fock-Feynman-Schwinger Representation (FFSR) [23], for the $2g$ Green’s function $\Pi(x, y)$ in the nonzero background

$$\Pi(x, y) = \int_0^\infty ds_1 \int_0^\infty ds_2 (Dz^{(1)})_{xy} (Dz^{(2)})_{xy} e^{-K_1 - K_2} W_\sigma(x, y) \quad (8)$$

where $K_i = \frac{1}{4} \int_0^{s_i} \left(\frac{dz_\mu^{(i)}}{dt} \right)^2 d\tau_i$, and W_σ is the Wilson loop with paramagnetic gluon spin insertions,

$$W_\sigma(x, y) = P \exp \left(ig \int_{C(x,y)} A_\mu dz_\mu \right) \exp \left(2ig \int_0^s F(z(\tau)) \right) d\tau. \quad (9)$$

Here $C(x, y)$ is the loop contour formed by the paths of two gluons from the point x to the point y . Averaging over the vacuum configurations one obtains $\bar{\Pi}(x, y)$, which is expressed only in terms of einbein parameters to be found from the $2g$ Hamiltonian [24, 25], (see Appendix 3 of [11] for details)

$$\bar{\Pi}(x, y) = \frac{1}{4(2\pi)^{5/2}} \int_0^\infty \int_0^\infty \frac{d\mu_1 d\mu_2 e^{-\frac{\mu_1 + \mu_2}{2} T}}{\tilde{\mu}^{3/2} \sqrt{T}} G(0, 0, T) \quad (10)$$

where we have defined $T \equiv |x - y|$, $\tilde{\mu} = \frac{\mu_1 \mu_2}{\mu_1 + \mu_2}$,

$$G(0, 0, T) = \langle 0 | e^{-H_{2g} T} | 0 \rangle = \sum_{n=0}^{\infty} |\psi_{2g}^{(n)}(0)|^2 e^{-E_{2g}^{(n)} T}. \quad (11)$$

Here H_{2g} is the $2g$ Hamiltonian with confinement and spin-dependent interaction, derived in [24, 25], and “0” refers to zero intergluon distance in the initial and final state.

The S -wave spectrum of the Hamiltonian to the lowest order in spin splittings is well known [24–26]

$$M_n^{(\mu_1, \mu_2)} = \frac{\mu_1 + \mu_2}{2} + \varepsilon_n(\tilde{\mu}), \quad \varepsilon_n(\tilde{\mu}) = (2\tilde{\mu})^{-1/3} \sigma_{\text{adj}}^{2/3} a(n), \quad a(n) \approx \left(\frac{3\pi}{2}\right)^{2/3} \left(n + \frac{1}{2}\right)^{2/3}. \quad (12)$$

Here $\sigma_{\text{adj}} = \frac{9}{4} \sigma_{\text{found}}$ is the gluonic string tension, and it is conceivable, that this gluonic string does not decay for $N_c \rightarrow \infty$, however even for finite N_c the main results are not sensitive to the high excitations, as will be seen, and hence to the string decay. Inserting $E_{2g}(n) = M_n$ from (12) and $|\psi_{2g}^{(n)}(0)|^2 = \frac{\sigma_{\text{adj}} \tilde{\mu}}{4\pi}$, [27], one obtains

$$\bar{\Pi}(x, y) = A \sum_n \int \frac{d\mu_1 d\mu_2}{\sqrt{\tilde{\mu} T}} e^{-M_n(\mu_1, \mu_2) T} \quad (13)$$

where $A = \frac{\sigma}{4(2\pi)^{7/2}}$.

Following [11], for large T one can do integration over $d\mu_1, d\mu_2$ using the steepest descent method, which yields stationary point $\mu_1^{(0)} = \mu_2^{(0)} = \frac{1}{4} \bar{M}_n$, where

$$\bar{M}_n = 4\sqrt{\sigma_{\text{adj}}} \left(\frac{a(n)}{3}\right)^{3/4} \quad (14)$$

and the resulting form of $\Pi(x, y)$ is

$$\bar{\Pi}_{IR}(x, y) = \frac{A\pi\sqrt{3}}{T^{3/2}} \sum_n \sqrt{\bar{M}_n} e^{-\bar{M}_n T}. \quad (15)$$

At small T one instead goes from the sum over n to the integral, which yields

$$\bar{\Pi}_{AF}(x, y) = \frac{1}{4(2\pi)^4 T^2} \int_0^\infty d\mu_1 \int_0^\infty d\mu_2 e^{-\frac{\mu_1 + \mu_2}{2} T} = \frac{1}{16\pi^4 T^4} \quad (16)$$

which reproduces the free result (3).

Let us now turn to the momentum space. The analysis of Polyakov in the free case can be written in the form

$$\frac{1}{g^2(Q^2)} = \frac{1}{g^2(\mu_0^2)} - \Pi_{\text{para}}(Q^2) + \Pi_{\text{dia}}(Q^2), \quad (17)$$

where

$$\Pi_{\text{para}}(Q^2) = 4N_c \Pi(Q^2), \quad \Pi_{\text{dia}}(Q^2) = \frac{N_c}{3} \Pi(Q^2), \quad (18)$$

and $\Pi(Q^2)$ in the free case is simply a scalar gluon loop, which after renormalization takes the form

$$\Pi_{\text{free}}(Q^2) = \int_{\mu_0} \frac{d^4 p}{(2\pi)^4 p^2 (p+Q)^2} = -\frac{1}{16\pi^2} \ln \left(\frac{Q^2}{\mu_0^2} \right). \quad (19)$$

Let us now turn to the case of perturbation theory in the confining vacuum. In this case two gluons in the loop form bound states, and we can use the spectrum, given in (14) for large n

$$\bar{M}_n = \frac{8\pi\sigma_a}{\sqrt{3}} \left(n + \frac{1}{2} \right) = m^2 + cn, \quad c = 4\pi\sigma_a \left(\frac{2}{\sqrt{3}} \right), \quad m^2 = \frac{4\pi\sigma_a}{\sqrt{3}}. \quad (20)$$

For the WKB spectrum in the linear potential $\sigma_a r$ one would obtain instead [28]

$$m_{\text{WKB}}^2 = 2\pi\sigma_a, \quad c_{\text{WKB}} = 4\pi\sigma_a \quad (21)$$

and we shall exploit these values (differing by 15% from m^2 and c respectively) in what follows.

In terms of the bound glueball states $\Pi(Q^2) \rightarrow \Pi_{\text{conf}}(Q^2)$ can be written as [29]

$$\Pi_{\text{conf}}(Q^2) = \sum_{n=0}^{\infty} \frac{f_n^2}{Q^2 + M_n^2} < \quad f_n^2 = \frac{M_n}{4\mu^2} |\psi_{2g}^{(n)}(0)|^2 = \frac{\sigma_a}{4\pi} \quad (22)$$

Replacing in (22) the sum over n by the integral and renormalizing the integral in the same way as in (19) one obtains

$$\Pi_{\text{conf}}(Q^2, \mu^2) = -\frac{1}{16\pi^2} \ln \frac{Q^2 + m_{\text{WKB}}^2}{\mu_0^2} \quad (23)$$

This latter form coincides at large Q^2 with the perturbation theory result

$$\Pi_{\text{conf}}(Q^2, \mu^2) \Big|_{Q^2 \rightarrow \infty} = \Pi_{\text{free}}(Q^2) = -\frac{1}{16\pi^2} \ln \left(\frac{Q^2}{\mu^2} \right) \quad (24)$$

Thus the charge evolution to the leading order in the confined vacuum can be written as

$$\frac{1}{g^2(Q^2)} = \frac{1}{g^2(\mu^2)} - \frac{11}{3} N_c \Pi_{\text{conf}}(Q^2, \mu^2). \quad (25)$$

One can see from (27), that for large $\frac{Q^2 + m^2}{\mu^2}$, $\Pi_{\text{conf}}(Q^2, \mu^2) < 0$ and the AF appears, $g^2(Q^2) < g^2(\mu^2)$.

For nonasymptotically large Q^2 , but still when $\frac{Q^2+m^2}{\mu^2}$ is large enough, the standard form of one-loop result for $\alpha_s(Q^2)$ using (23), (25) acquires the form

$$\alpha_s^{\text{conf}}(Q^2) = \frac{4\pi}{b_0 \ln \frac{Q^2+m^2}{\Lambda^2}}. \quad (26)$$

This form coincides with the one, proposed long ago in [12, 13], where m was associated with the effective mass of two gluons. As one can see, this notion of mass is in reality extended to the ground state mass of two gluons connected by the adjoint string, i.e. a ground-state glueball mass $M_{2g}(0^{++})$.

Numerically, however, $M_{2g}(0^{++})$ is large, from (21) $M_{2g}(0^{++}) = m_{\text{WKB}} = 1.6 \text{ GeV}$, which agrees with explicit calculations in [24, 25], while phenomenological estimate for m in the α_s , entering the static $Q\bar{Q}$ potential, is $m \approx 1 \text{ GeV}$ [15, 16]. In the next section we consider this situation in detail and shall find $m_{Q\bar{Q}}$ for the static potential.

3. ONE-LOOP EVOLUTION OF α_s FOR THE STATIC $Q\bar{Q}$ POTENTIAL

The purely perturbative derivation of static potential is given in [30]; for the case of confinement this situation was considered in detail in [11]. Below we shall give the main results of [11] and, as a new element, we estimate numerically the IR freezing mass m in Eq. (26) for the explicit case of the static $Q\bar{Q}$ potential, to be called $m_{Q\bar{Q}}$.

One starts with the Wilson loop with rectangular contour C of size $R \times T$ containing both nonperturbative confining background B_μ and valence gluons a_μ . Expanding in powers of (ga_μ) , one obtains a series of diagrams with valence gluon exchanges in the background field B_μ and after vacuum averaging

$$\langle W(B+a) \rangle_{B,a} = \exp(-V(R)T - \text{perimeter}) \quad (27)$$

one has terms

$$V(R) = V_q(R) + \alpha_s V_2(R) + \alpha_s^2 V_4(R) + \dots \quad (28)$$

Here $V_q(R) = \sigma_f R$ at large R , while $V_n(R), n \geq 2$, contains up to n gluons propagating inside the minimal surface S bounded by the contour C .

Thus $V_2(R)$ corresponds to the one-gluon exchange; while $V_4(R)$ contains a gluon loop (minus ghost loop) on the gluon propagator, a triangle vertex part and double gluon exchange and we take the limit $N_c \rightarrow \infty$, so that each gluon line is represented as a double fundamental

line. We also take the limit $T \rightarrow \infty$ to define static potential properly. In this case the change in the area of the minimal surface S due to gluon propagation is only due to inner closed loops in $V_4(R)$, while $V_2(R)$ and all single gluon lines are unaffected by confinement,

$$V_2(R) = -g^2 \frac{C_2(f)}{4\pi^2} \int_0^T \int_0^T \frac{dx_4 dy_4}{(x_4 - y_4)^2 + R^2} = -\frac{\alpha_s^{(0)} C_2(f) T}{R}. \quad (29)$$

We can write the contribution of $V_2 + V_4$ in the form

$$V_2(R) + V_4(R) = -\frac{C_2(f) \alpha_s^{(0)}}{R} (1 + \alpha_s^{(0)} f(R)), \quad (30)$$

where $f(R)$ was computed in case of no confinement in [11, 30]

$$f_0(R) = \frac{b_0}{4\pi} \ln \left(\frac{R}{\delta} \right)^2, \quad \delta \sim 1/\mu. \quad (31)$$

In the \overline{MS} scheme $f_0(R)$ was found to be [31]

$$f_0^{\overline{MS}}(R) = \frac{b_0}{4\pi} (\ln 9\mu^2 R^2) + 2\gamma_E + \frac{1}{\pi} \left(\frac{5}{12} b_0 - \frac{2}{3} N_c \right). \quad (32)$$

When confinement is included in the background, $f_{\text{conf}}(R)$ is expressed through the gluon selfenergy term $\bar{\Pi}(x - y)$, introduced in the previous section.

$$v_{\text{conf}}(R) \equiv \frac{f_{\text{conf}}(R)}{R} = \frac{\bar{b}_0}{4\pi^2} \int \frac{d^4 r \bar{\Pi}(r)}{|\mathbf{R} - \mathbf{r}|}, \quad \bar{b}_0 = \frac{11}{3} N_c. \quad (33)$$

In the momentum space the Fourier transform of the f_{conf}/R can be written as

$$\tilde{v}_{\text{conf}}(\mathbf{Q}) = \int d^3 R v_{\text{conf}}(R) e^{i\mathbf{Q}\mathbf{R}} = \frac{\bar{b}_0}{\pi} \frac{\Pi(\mathbf{Q})}{\mathbf{Q}^2}, \quad (34)$$

and using (23) this can be written as

$$\tilde{v}_{\text{conf}}(\mathbf{Q}) = -\frac{\bar{b}_0}{16\pi^3 \mathbf{Q}^2} \ln \frac{\mathbf{Q}^2 + m^2}{\mu^2} \quad (35)$$

Hence the total one-loop potential in momentum space has the form

$$\tilde{V}_2(\mathbf{Q}) + \tilde{V}_4(\mathbf{Q}) = -\frac{C_2(f) \alpha_s^{(0)}}{4\pi^2 \mathbf{Q}^2} \left(1 - \frac{\bar{b}_0}{4\pi} \alpha_s \ln \frac{\mathbf{Q}^2 + m_{Q\bar{Q}}^2}{\mu^2} \right). \quad (36)$$

It is now essential, that the string, connecting the gluons in the internal loop, is fundamental, and therefore

$$m_{Q\bar{Q}}^2 = 2\pi\sigma_f, \quad m_{Q\bar{Q}} = 1.06 \text{ GeV}. \quad (37)$$

This is important result, since the IR freezing of the gluon-exchange potential is essential for the quark model calculations, e.g. of hadron masses (see [15, 16, 32]), as well as in different QCD processes; it can also be tested on the lattice. In the next section we shall compare the result of (37) with other approaches.

4. RESULTS AND DISCUSSION

We have studied in previous sections the α_s renormalization with and without confinement in two different settings: the Polyakov background setting in section 2 and the $Q\bar{Q}$ interaction in section 3. We have found, that in the quenched ($N_c \rightarrow \infty$) case the one-loop α_s is given by the same equation in all cases

$$\alpha_s(Q^2) = \frac{4\pi}{\bar{b}_0 \ln \frac{Q^2+m^2}{\Lambda^2}}, \quad \bar{b}_0 = \frac{11}{3}N_c. \quad (38)$$

Here $m^2 = 0$ for the case of no confinement in both types of setting, confirming that the AF is a universal phenomenon. However, the IR mass m is not universal, it is $m^2 \equiv m_{gg}^2 = 2\pi\sigma_a$ in case of Polyakov background approach and $m^2 \equiv m_{Q\bar{Q}}^2 = 2\pi\sigma_f$ in case of the $Q\bar{Q}$ potential, and the latter can be written as

$$\tilde{V}_{Q\bar{Q}}(Q) = -4\pi C_2(f) \frac{\alpha_V(Q)}{Q^2}, \quad (39)$$

and $\alpha_V(Q)$ to one loop is the same, as in (38) with $\Lambda \rightarrow \Lambda_V$, while in the two-loop approximation can be written as [17, 18]

$$\alpha_V^{(2)}(Q) = \frac{4\pi}{\bar{b}_0 t_B} \left(1 - \frac{\bar{b}_1}{\bar{b}_0^2} \frac{\ln t_B}{t_B} \right) \quad (40)$$

with $t_B \equiv \ln \frac{Q^2+m^2}{\Lambda_V^2}$, $\bar{b}_1 = 102$, $\bar{b}_0 = 11$.

The coordinate-space representation $V_{Q\bar{Q}}(r)$ was studied in detail in [18] and it was shown, that to a reasonable accuracy (better than 10% for $r \geq 0.2$ fm) $\tilde{\alpha}_V(r)$ can be approximated by $\alpha_V(Q = 1/r)$, so that

$$V_{Q\bar{Q}}(r) = -\frac{C_2(f)}{r} \frac{4\pi}{\bar{b}_0 \ln \left(\frac{1/r^2+m^2}{\Lambda_V^2} \right)} \equiv -\frac{C_2(f)}{r} \tilde{\alpha}_V^{(1)}(r). \quad (41)$$

In the two-loop case $V_{Q\bar{Q}}(r)$ was computed in [17, 18] and is given by the same Eq.(41) where now the two-loop $\tilde{\alpha}_V^{(2)}(r)$ is [18]

$$\tilde{\alpha}_V^{(2)}(r) = \tilde{\alpha}_V^{(1)}(r) \left\{ 1 + B_1(r) \frac{\alpha_V^{(1)}(r)}{4\pi} + B_2(r) \left(\frac{\alpha_V^{(1)}(r)}{4\pi} \right)^2 \right\}, \quad (42)$$

where $B_1(r) = a_1 + 2\gamma_1(r)\bar{b}_0$, $B_2(r) = a_2 + 2\gamma_1(r)(\bar{b}_1 + 2\bar{b}_0 a_1) + \bar{b}_0^2 \gamma_2(r)$ and

$$\gamma_n(r) = \frac{2}{\pi} \int_0^\infty dx \frac{\sin x}{x} (\tilde{t}(x))^n, \quad \tilde{t}(x) = \ln \left(\frac{1+m^2 r^2}{x^2 + m^2 r^2} \right);$$

$$\bar{a}_1 = \frac{31}{3}, \quad \bar{a}_2 = \left[\frac{4343}{162} + 4\pi^2 - \frac{\pi^4}{4} + \frac{22}{3}\zeta(3) \right].$$

One can compare the one-loop results in p space (38), or two-loop result (40), and the corresponding x -space expressions (41) and (42) with lattice and experiment. Lattice data for $\alpha_V(r)$ from [33] were compared with $\tilde{\alpha}_V^{(2)}(r)$ in [17] and shown to be in good agreement in the region $0.04 \text{ fm} \leq r \leq 0.4 \text{ fm}$, measured in [33], while the purely perturbative $\alpha_{\text{pert}}^{(2)}(r)$ strongly deviates from lattice data already for $r > 0.08 \text{ fm}$. a similar good agreement can be deduced, comparing $\alpha_V^{(2)}(r)$ to the Schroedinger functional lattice method (second reference in [6]). On the phenomenological side, hadron spectra and especially fine-structure splittings are sensitive to the behavior of $\alpha_V(r)$. The saturated (frozen) value of $\alpha_V = \alpha_{\text{crit}}$ at large r was assumed in the detailed calculations in [32] and this value of α_{crit} agrees well with found in [17, 18]. Moreover, the analysis of the splittings between low-lying levels in bottomonium [15], yields $\alpha_{\text{crit}} = 0.58 \pm 0.02$ in striking agreement with [32].

A detailed analysis of bottomonium splittings in comparison with lattice data and experiment [16] proves, that the potential $V_{Q\bar{Q}}(r)$ (41) with $\tilde{\alpha}_V^{(2)}(r)$ obtained in the confining background perturbation theory with $m_{Q\bar{Q}} = 1 \text{ GeV}$ is in good agreement with experiment. This confirms the agreement between the calculations of the present paper and the physical reality.

Concluding one should stress the crucial role of gluon paramagnetism in creating the properties of AF and IR freezing. The correct sign of the logarithmic term in (26) is important for its Minkowskian extrapolation in the form of the sum of pole terms in the ψ -function, which in its turn correspond to correct physical poles in α_s , as shown in [21]. Moreover, as shown in [34], the gluon paramagnetism is responsible for the correct physical behavior of the field correlator $\langle F_{\mu\nu}^a(x) F_{\mu\nu}^a(x) \rangle$, implying nonzero and positive value of gluonic condensate.

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